Calc BC: Sec 9.3 Pg 500 # 1, 4, 5, 6, 8, 19, 22, 23, 27, 33

① $e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

$e^{2x} \approx 1 - 2x + \frac{4x^2}{2!} - \frac{8x^3}{3!} + \frac{16x^4}{4!}$

$f(0.2) \approx P_4(0.2) = 0.6704$ exact $= 0.670320046$

④ $\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

$\ln(1+x^2) \approx x^2 - \frac{x^4}{2}$

$f(0.2) \approx P_4(0.2) = 0.0392$ exact $= 0.039220713$

⑤ $\frac{1}{1-x} = 1 + x + x^2 + x^3 \dots$

$\frac{1}{(1-x)^2} = \frac{d}{dx}\left(\frac{1}{1-x}\right) = 1 + 2x + 3x^2 + 4x^3 + 5x^4$

$f(0.2) \approx P_4(0.2) = 1.56$ exact $= 1.5625$

⑥ $\sin x - x + \frac{x^3}{3!} = \cancel{-x} + \frac{x^3}{3!} + \cancel{x} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

$= \sum_{n=2}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

$\Rightarrow$

(8) $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^n \frac{(x)^{2n}}{(2n)!} + \dots$

$\cos(2x) = 1 - \frac{4x^2}{2} + \frac{16x^4}{24} - \dots + (-1)^n \frac{(2x)^{2n}}{(2n)!} + \dots$

$= 1 - 2x^2 + \frac{2}{3}x^4 - \dots$

$\frac{1}{2}(1+\cos(2x)) = \cos^2 x = 1 - x^2 + \frac{1}{3}x^4 - \frac{2x^6}{45} \dots + (-1)^n \frac{2^{n-1}\cdot x^{2n}}{(2n)!}$

(16) ~~want~~ $\cancel{\lim_{n\to\infty} R_n = 0}$

$\cancel{|R_n| = |f^{n+1}(t)|}$

(19) $\sin x \approx x - \frac{x^3}{3!} = P_3 = P_4$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{4}(x) = \sin x$

$f^{5}(x) = \cos x$

$|f^{5}(x)| = |\cos x| \le 1$

$\rightarrow |R_4(x)| \le \frac{1\cdot|x|^5}{5!} < 5 \times 10^{-4}$

$|x| < \sqrt[5]{.06}$

$|x| < .5697$

$[-0.1, 0.1]$

$\ln(1+x) \approx x - \frac{x^2}{2} = P_2(x)$

$f(x) = \ln(1+x)$

$f'(x) = \frac{1}{1+x}$

$f''(x) = \frac{-1}{(1+x)^2}$

$f'''(x) = \frac{2}{(1+x)^3}$

$f'''(x)$ is decreasing

so max is @ $\underline{\text{left}}$

endpoint at $x = -0.1$

$|f^3(t)| \le \frac{2}{(1+(-0.1))^3} = M = \frac{2000}{729}$

$|R_2| \le \frac{2000}{729} \cdot \frac{|x|^3}{3!} \quad x = .1$

$< 4.5725 \times 10^{-4}$

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(23) $e^x \approx 1 + x + \frac{x^2}{2!}$ estimate error for $|x| < 0.1$

$-0.1 < x < 0.1$

$M = |f'''(t)| < e^{.1}$

$|R_2| = e^{.1} \cdot \frac{|x|^3}{3!}$

$= e^{.1} \frac{|.1|^3}{3!}$

$= 1.842 \times 10^{-4}$

$P_n = \ldots \frac{f^n(x)}{n!}(x-a)^n$

$$|R_n(x)| \leq \frac{f^{n+1}(c)}{(n+1)!}(x-a)^{n+1}$$

or

$\frac{M}{(n+1)!} \cdot (x-a)^{n+1}$ → Max value

<u>Ex.</u> 3rd degree of sin x

$P_3(x) = x - \frac{x^3}{3!}$

Max error

$0 \leq x \leq .1$

$|R_n| = \frac{f^4(c)}{4!}(x-a)^4$

centered @ $x=0$

$\Rightarrow a=0$

$f(x) = \sin x$

$f'(x) =$

$f''(x) =$

$f'''(x) =$

$f^4(x) = \sin x$

$$|R_n(x)| \leq \frac{\sin(x)}{4!}(x)^4$$

$$\leq \frac{\sin(.1)}{4!}(.1)^4$$

$$\leq 4.1597 \times 10^{-7}$$

(22) $f(x) = \sqrt{1+x}$ $\qquad P_1(x) = 1 + \frac{x}{2}$ $\qquad |x| < 0.01$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $\qquad -.01 < x < .01$

$|f''(x)| = \left| -\frac{1}{4}(1+x)^{-3/2} \right|$ has max value @ $x = -.01$

$= .2538$ $\qquad m = f^{(n+1)}(b)$

$$|R_2(x)| \le \frac{m|x|^2}{2!} = \frac{m|.01|^2}{2}$$

$\le 1.269 \times 10^{-5}$

(27) $f(x) = \ln(\cos(x))$, $f(0) = 0$

$f'(x) = \frac{1}{\cos x} \cdot -\sin x$

$= -\tan x$ $\qquad f'(0) = 0$

$f''(x) = -\sec^2 x$ $\qquad f''(0) = -1$

a) $y - 0 = x - 0$

$y = x$

b) $P_2(x) = 0 - \frac{0}{1!}x - \frac{1}{2!}x^2$

$= -\frac{1}{2}x^2$

(33) $e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$ $\quad -.1 < x < .1$

$|f^{4}(.1)| \le M = e^{.1}$

$|R_3(x)| = \frac{e^{.1}|.1|^4}{4!} = 4.605 \times 10^{-6}$